Modeling and Solving AP Location and Frequency Assignment for Maximizing Access Efficiency in Wi-Fi Networks

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Abstract

In this paper, we present optimization approaches for Access Point (AP) location and frequency assignment, two major planning tasks in deploying Wi-Fi networks. Since APs are relatively cheap, the major concern is network performance. We consider a performance metric, referred to as access efficiency, that captures key aspects of how user devices share access to the wireless medium. We propose a two-step approach to deal with AP location and frequency assignment in maximizing access efficiency. A novelty of our modeling approach is to estimate, in the first step of AP location, the impact of expected frequency availability on frequency assignment. For each of the two steps we derive hyperbolic formulations and their linearizations, and we propose a promising enumerative formulation. Sample results are reported to show the applicability of the approach.

1   Introduction

A Wi-Fi network consists in a set of Access Points (APs) connected to a wired backbone. Two major planning tasks are to decide, given a set of *candidate sites* (CSs) and a set of frequencies, where to install APs and which frequency to assign to each of them. A typical constraint is to require coverage for a set of *test points* (TPs) representing user locations, as in the classical Set Covering Problem. As APs are relatively cheap. The major concern is network performance. If a TP is covered by multiple APs, a user device at the TP will select one of them and *associate* to it, choosing one of the possible transmission rates (e.g. 6, 9, 12, 18, 24, 36, 48, and 54 Mbps in IEEE 802.11g). As rate selection depends on signal quality, devices typically associate to the AP providing the highest signal quality, and thus rate.

The Wi-Fi medium access mechanism (CSMA/CA) is a carrier sense protocol: Before transmitting, a device must sense the channel as idle. In IEEE 802.11b and 802.11g there are up to 14 overlapping channels, but due to inter-channel interference it is a common practice to restrict channel assignment to the three non-overlapping channels 1, 6 and 11. Under this assumption, medium contention can take place only between devices operating on the same frequency. We use the term *direct interference* to refer to a medium contention occurring between two devices because their signals directly reach each other. *Indirect interference* refers to a signal collision at one of the associated APs. One example is the *hidden terminal* scenario: Two devices that cannot sense each other wish to transmit to the same AP. Carrier sensing allows both transmissions, resulting in a collision at the AP. Note that even if the two devices are transmitting to different APs a collision may still occur, if the transmission from a device to its AP reaches also the second AP, which is receiving from the other device.

To summarize, two user devices working on the same frequency interference with each other if they are direct interferers, or if (at least) one is associated to an AP reached by the signal of the other. In Section 2 we present a performance metric for devices, referred to as *access efficiency*, defined as the transmission rate scaled by the probability of successful transmission, determined by the number of direct and indirect interferers. For simplicity, direct interference will not be considered in this work. Its inclusion into models and algorithms is straightforward (see [6]).

A simplified version of this metric has been studied and validated in [4], and theoretically and algorithmically investigated in [2, 3]. This paper extends these works to consider TP-AP association, rates, and multiple frequencies. In Sections 3 and 4 we present integer programming models for access efficiency maximization in AP location and frequency assignment, and discuss solution approaches. The difference of this modeling approach with respect to others (see e.g. [1, 7, 8]) is the aspect that the performance metric originates from the Wi-Fi medium access scheme. For a discussion on alternative modeling and solution approaches for Wi-Fi network design, see [5] and the references therein.

2   Access Efficiency and Wi-Fi Planning

The sets of TPs and CSs are denoted by I and J respectively, and the set of frequencies by F. We use "AP j" to refer to an AP installed at CS j, and we refer equivalently to "TPs" or "users". The set of TPs covered by AP j and the set of APs covering TP i are denoted respectively by $I_j \subseteq I$ and $J_i \subseteq J$. The set of TPs covered by a subset $S \subseteq J$ of CSs is denoted by $I(S) = \bigcup_{j \in S} I_j$, and if $I(S) = I$ the set S is said to be a *cover*. The set of TPs $N_i = I(J_i)$ sharing a covering AP with TP i is referred to as the set of *neighbors* of i. Figures 1(a) and 1(b) illustrate the coverage and neighborhood relations.

A design solution is a pair (S, f), where $S \subseteq J$ is a cover and $f : S \to F$ is the frequency assignment. To model TP-AP association, we introduce for each TP i a total order $>_i$ on J_i, where $j >_i k$ means that at TP i the signal of AP j is stronger than that of AP k. Given a cover S, we denote by $a_i = a_i(S)$ the AP to which TP i associate. This is the (unique) AP $j \in S \cap J_i$ for which $j >_i k$ for every $k \in S \cap J_i \setminus \{j\}$. If TP i associates to AP j (shortly, if $a_i = j$) then no AP covering i with signal better than j is installed. We denote by $J_{ij} = \{k \in J_i : j >_i k\}$ the set of APs covering i and compatible with the association $a_i = j$, and by $\tilde{J}_{ij} = \{k \in J_i : k >_i j\}$ those that are not compatible. Moreover, we define the sets of TPs $N_{ij}^{\mathrm{CS}} = I_j \setminus \{i\}$ and $N_{ij}^{\mathrm{SF}} = I(J_{ij}) \setminus I_j$. These notations are illustrated in Figure 1(c).

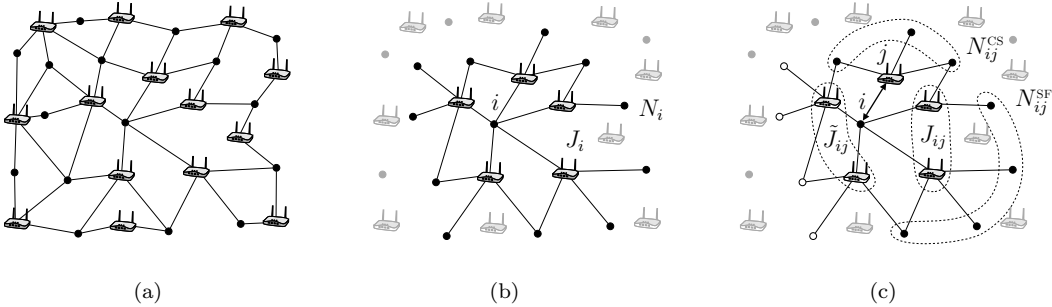(a)   (b)   (c)

Figure 1: (a) An instance, (b) covering APs and neighbors of a TP, (c) association-related sets.

A design solution (S, f) induces a frequency assignment $f : I(S) \to F$ to the covered TPs, with $f(i) = f(a_i)$. Given a design solution (S, f), the set of interferers to TP i can then be defined as $\Phi_i(S, f) = \{h \in N_i : f(i) = f(h) \ \wedge \ (a_i \in J_h \ \vee \ a_h \in J_i)\}$. Denoting by Γ_{ij} the data rate between TP i and AP j, the access efficiency metric, under uniform traffic and equal access, reads:

$$e(S, f) = \sum_{i \in I(S)} \frac{\Gamma_{ia_i}}{1 + |\Phi_i(S, f)|}. \tag{1}$$

The **Wi-Fi Planning Problem (WPP)** consists in finding a design solution (S, f) maximizing $e(S, f)$.

WPP is NP-hard, even in the special cases $|F| = 1$ (all APs working at the same frequency) and $|F| = |S|$ (complete separation). Moreover, the frequency assignment problem, obtained from WPP by fixing the AP location S, is also NP-hard. We refer to [6] for complete proofs. In this paper we propose a decomposition approach to deal with AP location and frequency assignment in two steps. A novelty of our approach is that the two tasks are not completely separated, as in the AP location step we try to estimate the impact of frequency availability on access efficiency.

3 AP Location

Consider the extreme case of planning AP location with a single frequency (SF), i.e., $|F| = 1$. The set of interfering TPs of TP i, given a cover S, is $\Phi_i^{\mathrm{SF}}(S) = \{h \in N_i : a_i \in J_h \ \vee \ a_h \in J_i\}$. The opposite assumption is that there will be enough frequencies to provide a frequency assignment in which two TPs interfere if and only if they are associated to the same AP, e.g., $|F| = |S|$. Note, however, that the actual number of frequencies required for complete separation (CS) is typically much smaller than $|S|$, because frequencies can be reused. The set of interfering TPs of TP i is then $\Phi_i^{\mathrm{CS}}(S) = \{h \in N_i : a_i = a_h\}$. The access efficiency of **Single Frequency AP location** (SFAP) and **Complete Separation AP location** (CSAP) are respectively

$$e^{\mathrm{SF}}(S) = \sum_{i \in I(S)} \frac{\Gamma_{ia_i}}{1 + |\Phi_i^{\mathrm{SF}}(S)|} \qquad \text{and} \qquad e^{\mathrm{CS}}(S) = \sum_{i \in I(S)} \frac{\Gamma_{ia_i}}{1 + |\Phi_i^{\mathrm{CS}}(S)|}. \tag{2}$$

For real-life Wi-Fi deployment, the SFAP assumption is too conservative, while the CSAP one is too optimistic. We therefore use a convex combination of the two. Consider a TP $i \in I$ and a cover S. In SFAP a TP $h \in N_i$ interferes with i if $a_i \in J_h$ or $a_h \in J_i$, while in CSAP it interferes if $a_i = a_h$. Clearly $\Phi_i^{\mathrm{CS}}(S) \subseteq \Phi_i^{\mathrm{SF}}(S)$. Users in $\Phi_i^{\mathrm{CS}}(S)$ always interfere with user i, while interference with users in $\Phi_i^{\mathrm{SF}}(S) \setminus \Phi_i^{\mathrm{CS}}(S)$ depends on frequency assignment. We consider the convex combination $\alpha|\Phi_i^{\mathrm{SF}}(S)| + (1 - \alpha)|\Phi_i^{\mathrm{CS}}(S)|$, where $\alpha \in [0, 1]$ represents the likelihood that users in $\Phi_i^{\mathrm{SF}}(S) \setminus \Phi_i^{\mathrm{CS}}(S)$ will interfere with i under optimal frequency assignment. The objective of the **Partial Separation AP location Problem** (PSAP), which generalizes both SFAP and CSAP, is to maximize

$$e^{\mathrm{PS}}(S, \alpha) = \sum_{i \in I(S)} \frac{\Gamma_{ia_i}}{1 + \alpha|\Phi_i^{\mathrm{SF}}(S)| + (1 - \alpha)|\Phi_i^{\mathrm{CS}}(S)|} = \sum_{i \in I(S)} \frac{\Gamma_{ia_i}}{1 + \alpha|\Phi_i^{\mathrm{SF}}(S) \setminus \Phi_i^{\mathrm{CS}}(S)| + |\Phi_i^{\mathrm{CS}}(S)|}. \tag{3}$$

An Integer Hyperbolic Model. Using the binary variables $x_j, j \in J$ for AP location, $l_{ij}, i \in I, j \in J_i$ for TP-AP association, and $y_{ih}, i \in I, h \in N_i, i \neq h$ for representing whether or not two TPs interfere, we obtain a nonlinear integer model of PSAP in which the objective functions is a hyperbolic sum.

$$\max \quad \sum_{i \in I} \sum_{j \in J_i} \frac{\Gamma_{ij} l_{ij}}{1 + \alpha \sum_{h \in N_i} y_{ih} + (1 - \alpha) \sum_{h \in N_i} l_{hj}} \tag{4}$$

$$\text{(PSAP-H)} \qquad s.t. \quad \sum_{j \in J_i} l_{ij} = 1 \qquad\qquad\qquad i \in I \tag{5}$$

$$l_{ij} \leqslant x_j \qquad\qquad\qquad i \in I, j \in J_i \tag{6}$$

$$x_j + \sum_{k \in J_{ij}} l_{ik} \leqslant 1 \qquad\qquad i \in I, j \in J_i \tag{7}$$

$$y_{ih} \geqslant \sum_{j \in J_i \cap J_h} l_{ij} \qquad\qquad i \in I, h \in N_i \tag{8}$$

$$y_{ih} = y_{hi} \qquad\qquad\qquad i \in I, h \in N_i, i < h \tag{9}$$

$$x_j, l_{ij}, y_{ih} \in \{0, 1\}.$$

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The meaning of (5) and (6) is obvious. A constraint of (7) states that, if AP j is installed and covers i, then TP i will not be associated to any AP covering i with weaker signal. By constraints (8), two TPs i and h interfere ($y_{ih} = 1$) if at least one of them associates to an AP covering the other. Constraints (9) state the symmetry of the interference relation (half of the y variables can thus be removed).

A natural approach to solving PSAP-H is to derive an integer linear reformulation. For $i \in I$ and $j \in J_i$, we introduce a new variable c_{ij} to represent the corresponding hyperbolic term in (4). The new objective function is thus $\max \sum_{i \in I} \sum_{j \in J_i} c_{ij}$. The value of c_{ij} is defined by the nonlinear constraint $c_{ij} + \alpha \sum_{h \in N_i} c_{ij} y_{ih} + (1-\alpha) \sum_{h \in N_i} c_{ij} l_{hj} = \Gamma_{ij} l_{ij}$. We can then linearize $c_{ij} y_{ih}$ and $c_{ij} l_{hj}$ with standard techniques. Consider for example $c_{ij} y_{ih}$. We introduce a new variable $z_{ihj} = c_{ij} y_{ih}$. As c_{ij} represents access efficiency, we can derive lower and upper bounds of the value of c_{ij}, denoted respectively $\underline{c}_{ij}$ and $\bar{c}_{ij}$. Then $z_{ihj} = c_{ij} y_{ih}$ can be defined by the inequalities $c_{ij} - \bar{c}_{ij}(y_{ih} - 1) \leqslant z_{ihj} \leqslant c_{ij} + \underline{c}_{ij}(y_{ih} - 1)$ and $\underline{c}_{ij} y_{ih} \leqslant z_{ihj} \leqslant \bar{c}_{ij}$. The term $c_{ij} l_{hj}$ can be linearized in the same way.

We remark that alternative hyperbolic formulations can be obtained by changing the definition of the y-variables (e.g., define y-variables for $i \in I$ and $h \in \Phi_i^{\mathrm{SF}}(S) \setminus \Phi_i^{\mathrm{CS}}(S)$). These formulations perform similar to PSAP-H from a computational viewpoint.

An Enumerative Integer Programming Model. A drawback of PSAP-H is that the continuous relaxation is very weak. A novel integer linear model can be derived by enumerating interference scenarios for each TP. Recall that J_{ij} and N_{ij}^{SF} are the sets of potential interferers in CSAP and SFAP, respectively (Figure 1(c)). We define an *interference scenario* of TP i as a tuple $s = (j_s, H_s, U_s)$, where $j_s \in J_i$ is the AP to which i associates, $H_s \subseteq N_{ij_s}^{\mathrm{CS}}$ is the set of users associated to j_s, and $U_s \subseteq N_{ij_s}^{\mathrm{SF}}$ is the set of users associated to some AP in J_{ij_s} but not covered by j_s. We denote by $\mathcal{S}_i$ the set of all interference scenarios of TP i. Defining new binary variables $w_{is}, i \in I, s \in \mathcal{S}_i$ to identify interference scenario, and reusing the x- and l-variables in PSAP-H, we obtain the following integer linear formulation.

$$\max \quad \sum_{i \in I} \sum_{s \in \mathcal{S}_i} \frac{\Gamma_{ij_s}}{1 + \alpha(|U_s| + |N_{ij_s}^{\mathrm{CS}}|) + (1-\alpha)|H_s|} w_{is}$$

(PSAP-L) $\quad s.t. \quad (5), (6), (7)$

$$\sum_{s \in \mathcal{S}_i : j = j_s} w_{is} = l_{ij} \qquad\qquad i \in I, j \in J_i \qquad (10)$$

$$\sum_{s \in \mathcal{S}_i : h \in U_s} w_{is} + \sum_{j \in J_i \cap J_h} l_{ij} = \sum_{s \in \mathcal{S}_h : i \in U_s} w_{hs} + \sum_{j \in J_i \cap J_h} l_{hj} \qquad i \in I, h \in N_i \qquad (11)$$

$$\sum_{s \in \mathcal{S}_i : j = j_s, h \in H_s} w_{is} \leqslant l_{hj} \qquad\qquad i \in I, j \in J_i, h \in N_{ij}^{\mathrm{CS}} \,(12)$$

$$\sum_{s \in \mathcal{S}_i : j = j_s, h \in H_s} w_{is} \geqslant l_{ij} + l_{hj} - 1 \qquad\qquad i \in I, j \in J_i, h \in N_{ij}^{\mathrm{CS}} \,(13)$$

$$x_j, l_{ij}, w_{is} \in \{0, 1\}.$$

The correctness of (10) is apparent. To see the correctness of (11), let us consider it case by case. For simplicity, let W_i, L_i, W_h and L_h represent the four summations in (11) (in the order they appear). In the first case, each of TPs i and h is associated to an AP covering also the other TP (i.e., $a_i, a_h \in J_i \cap J_h$). Then $L_i = L_j = 1$, and $W_i = W_h = 0$. Note that also $W_i = W_h = 1$ is a feasible but non-optimal assignment. In the second case, $a_i \notin J_h$, and $a_h \in J_i$ (or vice versa). Then $L_i = 0$ and $L_h = 1$, and necessarily $W_i = 1$ and $W_h = 0$. The last case is when $a_i, a_h \notin J_i \cap J_h$, meaning that $L_i = L_h = 0$, and thus $W_i = W_h = 0$. Again, $W_i = W_h = 1$ is feasible but non-optimal. Constraints (12) and (13) are the linearization of the bilinear constraint $\sum_{s \in \mathcal{S}_i : j = j_s, h \in H_s} w_{is} = l_{ij} l_{hj}$, which corresponds to the definition of interference in CSAP. Note that PSAP-L can also be obtained with Dantzig-Wolfe decomposition from an appropriate reformulation of PSAP-H.

We empirically observe that the continuous relaxation of PSAP-L yields much shaper bounds than that of the linearization of PSAP-H. For small or sparse instances, one can generate the interference scenarios in advance and apply a standard solver to PSAP-L. For large-scale instances, the exponential number of w-variables can be handled solving the LP relaxation of PSAP-L with column generation techniques. We remark that the pricing problem decomposes by TP, and for each TP i the optimal value (i.e., highest reduced cost of w_{is} among all $s \in \mathcal{S}_i$) can be found in polynomial time. Moreover, combining column generation with branching over the l-variables leads to a branch-and-price algorithm for PSAP-H. We omit the details of the algorithm because of lack of space.

4 Frequency Assignment

Given a solution $S \subseteq J$ of the AP location problem, the **Wi-Fi Frequency Assignment Problem** (WFAP) amounts to finding a frequency assignment $f : S \to F$ that maximizes the efficiency $e(S, f)$. As S is given, the association between TPs and APs is fixed. We assume that for each AP $j \in S$ there is at least one TP $i \in I_j$ for which $a_i = j$ (as otherwise the AP can be removed).

A fixed association corresponds to a specific scenario $s \in \mathcal{S}_i$ for each TP i, which we denote by $s_i = (a_i, H_i, U_i)$, where $H_i = \{h \in N_i : a_h = a_i\}$ are the neighbors of i associated to a_i and $U_i = \{h \in N_i \setminus I_{a_i} : a_h \in J_i\}$ are the neighbors of i that are not covered by a_i and are associated to some AP a_h covering i. Let us also denote by $\overline{U}_i = U_i \cup (I_{a_i} \setminus H_i \setminus \{i\})$ the set of neighbors of i that are associated to some AP $a_h \neq a_i$ covering i. Note that $\overline{U}_i$ defines the set of those TPs that may or may not interfere with i, depending on the frequency assignment. The set $\Phi_i(S, f)$ of interfering TPs for a given TP i can then be more explicitly defined as $\Phi_i(S, f) = H_i \cup \{h \in \overline{U}_i : f(i) = f(h)\}$, where the union is disjoint by definition. Introducing the frequency assignment variables x_{jf} ($x_{jf} = 1$ if $f(j) = f$, and 0 otherwise) and reusing y-variables, WFAP can be formulated as follows:

$$\max \quad \sum_{i \in I} \frac{\Gamma_{ia_i}}{1 + |H_i| + \sum_{h \in \overline{U}_i} y_{ih}}$$

$$\text{(WFAP-H)} \qquad s.t. \quad \sum_{f \in F} x_{jf} = 1 \qquad\qquad j \in S \qquad\qquad\qquad (14)$$

$$y_{ih} = \sum_{f \in F} x_{a_i f} x_{a_h f} \qquad i \in I, h \in \overline{U}_i \qquad\qquad (15)$$

$$x_{jf}, y_{ih} \in \{0, 1\}. \qquad\qquad\qquad\qquad (16)$$

An alternative hyperbolic formulation, based on an implicit partitioning of the APs into $|F|$ sets, can be obtained introducing variables v_{jk} ($v_{jk} = 1$ if $f(j) = f(k)$, and 0 otherwise) for all $(j, k) \in S^2$, where S^2 is the set of all ordered pairs (j, k) with $j, k \in S$ and $j < k$. Consider the case $|F| = 3$ (a similar formulation can be derived also for $|F| = 2$). As it is easy to note that $y_{ih} = v_{a_i a_h}$, we can write the following equivalent formulation:

$$\max \quad \sum_{i \in I} \frac{\Gamma_{ia_i}}{1 + |H_i| + \sum_{h \in \overline{U}_i} v_{ih}}$$

$$\text{(WFAP-H}_2\text{)} \qquad s.t. \quad \sum_{(j,k) \in T^2} v_{jk} \geqslant 1 \qquad T \subseteq S : |T| = 4 \qquad\qquad (17)$$

$$v_{jk} \geqslant v_{jl} + v_{kl} - 1 \qquad (j, k), (k, l) \in S^2 \qquad\qquad (18)$$

$$v_{kl} \geqslant v_{jk} + v_{jl} - 1 \qquad (j, k), (k, l) \in S^2 \qquad\qquad (19)$$

$$v_{jl} \geqslant v_{jk} + v_{kl} - 1 \qquad (j, k), (k, l) \in S^2 \qquad\qquad (20)$$

$$v_{jk} \in \{0, 1\} \qquad\qquad (j, k) \in S^2,$$

where $v_{ih} = v_{a_i a_h}$ if $(a_i, a_h) \in S^2$, and $v_{ih} = v_{a_h a_i}$ if $(a_h, a_i) \in S^2$.

An Enumerative Integer Programming Model. In order to define the objective function contribution for i, we need to know which TPs in $\overline{U}_i$ do interfere with i, that is, which APs in $C_i = \{j \in S : j = a_h \text{ for some } h \in \overline{U}_i\}$ operate on the same frequency of a_i. By enumerating on the set $\mathcal{A}_i = \{A \subseteq C_i\}$ of all subsets of C_i we can derive the following integer linear formulation.

$$\max \quad \sum_{i \in I} \sum_{A \in \mathcal{A}_i} \frac{\Gamma_{ia_i}}{1 + |H_i| + |\overline{\overline{U}}_i \cap I(A)|} w_{iA}$$

$$\text{(WFAP-L)} \qquad s.t. \quad (17), (18), (19), (20)$$

$$v_{a_i k} = \sum_{A \in \mathcal{A}_i : k \in A} w_{iA} \qquad i \in I, k \in C_i \qquad (21)$$

$$v_{jk} \in \{0,1\}, w_{ia} \in \{0,1\},$$

It is interesting to remark that the size of WFAP-L can be reduced by exploiting three-coloring heuristics (assuming three frequencies) on an appropriate *AP overlap graph* of the instance, of which the node set is S and the edge set is defined as $E = \{\{j,k\} \subseteq S : a_i \in \{j,k\} \text{ for some } i \in I_j \cap I_k\}$. If the overlap graph can be three-colored then it is possible to find a frequency assignment f for which $e(S,f) = e^{\text{CS}}(S)$. Although three-coloring is an NP-complete problem, it can be decided in polynomial time for many classes of graphs. Moreover, there are many polynomial algorithms providing a partial three-coloring, thus allowing size reduction of WFAP-L.

5 Sample results and concluding remarks

In this section we report on some preliminary experiments carried out on random 2D instances, with both isotropic propagation (where the coverage area of an AP is a disks) and random anisotropic propagation (see [3] for a description of the instances). In Table 1 we show the results obtained with four sample anisotropic instances with 50 CSs and 100 TPs. For each instance problem PSAP is solved for five different values of the parameter α, between 0.0 (CSAP) and 1.0 (SFAP). For each resulting AP location solution, problem WFAP is solved for both $|F| = 2$ and $|F| = 3$, to derive a complete network design. Problem PSAP is solved to optimality with three different formulations: two compact linearizations (PSAP-H$_1$ and PSAP-H$_2$), derived from PSAP-H with standard techniques as mentioned in Section 3, and the enumerative linearization PSAP-L. The solution of WFAP is not as critical as that of PSAP, and we do not discuss here the computational time needed. For a comparison of different solution methods for WFAP the reader is referred to [6]. Computational times reported in Table 1 are in seconds on a 1281 MHz SUN UltraSPARC-IIIi with 16 GB of RAM.

α	PSAP-H$_1$ time (sec)	PSAP-H$_2$ time (sec)	PSAP-L time (sec)	PSAP (o.f.)	WFAP $|F|=2$ (o.f.)	WFAP $|F|=3$ (o.f.)	α	PSAP-H$_1$ time (sec)	PSAP-H$_2$ time (sec)	PSAP-L time (sec)	PSAP (o.f.)	WFAP $|F|=2$ (o.f.)	WFAP $|F|=3$ (o.f.)
			INSTANCE 1							INSTANCE 3			
0.0	6.27	14.10	0.11	742.26	571.25	711.70	0.0	2.92	12.46	0.08	725.61	574.70	681.31
0.2	119.82	326.82	248.84	469.01	571.25	711.70	0.2	34.07	135.82	11.50	476.55	575.41	695.45
0.4	632.69	966.48	330.35	367.91	576.50	689.06	0.4	140.30	385.08	14.34	374.58	559.94	663.66
0.6	2430.33	2200.62	286.98	317.00	544.76	633.78	0.6	466.59	1596.62	15.08	315.58	559.61	650.93
0.8	6299.12	4440.09	380.36	286.11	508.66	581.70	0.8	1712.76	1505.59	14.46	277.06	536.88	606.36
1.0	13580.43	163.99	55.59	265.75	484.82	529.47	1.0	856.31	70.44	3.47	251.61	485.71	512.79
			INSTANCE 2							INSTANCE 4			
0.0	4.84	9.05	0.07	711.13	599.74	700.38	0.0	8.55	12.26	0.08	720.91	596.81	720.86
0.2	24.52	104.43	41.71	481.48	596.18	695.38	0.2	41.88	201.82	53.03	499.52	598.68	712.36
0.4	76.83	199.73	42.05	379.37	596.18	695.38	0.4	140.92	527.72	74.51	402.39	596.55	706.36
0.6	396.98	1001.74	42.90	318.82	589.05	679.08	0.6	925.80	2284.93	72.39	342.25	596.55	696.70
0.8	8069.79	14364.77	55.26	279.32	510.95	565.56	0.8	1829.28	6506.13	82.61	304.99	518.13	590.03
1.0	1423.05	164.28	6.52	256.98	469.76	486.35	1.0	1614.43	363.25	7.05	284.37	470.91	506.31

Table 1: Sample experimental results on random 2D anisotropic instances with 50 CSs and 100 TPs.

The direct linearization of WPP cannot be solved for these instances within days of computation, while the direct linearizations PSAP-H$_1$ and PSAP-H$_2$ of PSAP still remain manageable, even if they show a strong performance degeneration towards larger values of the parameter α, requiring up to four hours of CPU time. In comparison, the time needed to solve the enumerative formulation PSAP-L is by far smaller, at most seven minutes, and shows a uniform behavior for different values of α. Note that the much better performance showed for $\alpha = 0$ and $\alpha = 1$ by all the solution methods is due to the simpler structure of CSAP and SFAP, which the formulations can advantageously use.

For these instances the best performance is obtained by choosing a small α in the AP location phase, i.e., assuming that close-to complete separation can be achieved by frequency assignment. With three frequencies, $\alpha \leqslant 0.2$ gives the highest efficiency, while with two available frequencies the optimal value of α tends to increase slightly.

Concluding remarks. We have presented a two-phase approach to Wi-Fi planning, where in the AP location phase one can take into account the estimated impact of frequency availability on an optimal frequency assignment. For each phase we derive hyperbolic formulations and the corresponding linearizations, and a promising enumerative formulation approach. The enumerative formulations allow for exact algorithms using branch and price, of which the implementation is underway. More extensive experiments and result analysis will be carried out in forthcoming work.

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